Solving Quadratic Equations without Factoring

Recall: To solve by factoring,
(1) collect all terms on one side of equal sign
(2) factor the expression
(3) use (a)(b) = 0 to state a = 0 or b = 0

Consider this example:

$$x^2 - 12x + 32 = 0$$
$$(x-8)(x-4) = 0$$

$$x-8=0 \qquad x-4=0$$
$$x=8 \qquad x=4$$

4.9 (6.4)-Solving Quadratic Equations without Factoring

Vertex form can also be very useful for solving a quadratic equation.

finding roots

(a) Write $y = x^2 - 12x + 32$ in vertex form

$y = x^2 - 12x + 32$
$y = x^2 - 12x + 36 - 36 + 32$
$y = (x-6)^2 - 4$

b) Solve for $y = 0$

$0 = (x-6)^2 - 4$
$\sqrt{4} = \sqrt{(x-6)^2}$
$\pm 2 = x - 6$
$6 \pm 2 = x$
$x = 6 \pm 2$
$x_1 = 4$
$x_2 = 8$

In some cases, one may be simpler than the other.

Ex.2 Write in factored & vertex form, then choose which to use for solving.

(a) $x^2 + 3x - 4 = 0$

$(x-1)(x+4) = 0$
$x = 1$ or $x = -4$
AOS $= \frac{-4+1}{2}$
$x = -1.5$
Sub $x = -1.5$
$= (1.5-1)(-1.5+4)$
$= (-2.5)(2.5)$
$= -6.25$
$y = (x+1.5)^2 - 6.25$

$x^2 + 3x - 4 = 0$
$= x^2 + 3x + 2.25 - 2.25 - 4$
$y = (x+1.5)^2 - 6.25$
Set $y = 0$
$0 = (x+1.5)^2 - 6.25$
$\sqrt{6.25} = \sqrt{(x+1.5)^2}$
$\pm 2.5 = x + 1.5$
$-1.5 \pm 2.5 = x$
$x_1 = 1$
$x_2 = -4$

Ex.2 Write in factored & vertex form, then choose which to use for solving.

(b) $x^2 - 9 = 7$

$x^2 - 9 - 7 = 0$
$x^2 - 16 = 0$
$(x-0)^2 - 16$
factor
$(x+4)(x-4)$
$y = 0$
$x^2 - 16 = 0$
$\sqrt{x^2} = \sqrt{16}$
$x = \pm 4$

If factoring is not possible use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

which is derived from completing the square (p.337-338).

Note: To use the quadratic formula, the equation must be in standard form, $ax^2 + bx + c = 0$. standard form

The '$\pm$' symbol means there are two solutions.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Assigned Work:

p.343 # 1ad, 3, 4bdf, 5ace, 9ad, 10d, 14, 19*

Ex.3 Solve using the quadratic formula.

a) $x^2 - 4x - 3 = 0$ a b c

$x_1 \doteq 4.65$
$x_2 \doteq -0.65$
roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

b) $x^2 - 2x - 5 = 0$ a b c

$x_1 \doteq$ $x_2 \doteq$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{24}}{2}$$

$\frac{2+\sqrt{24}}{2}$ $\frac{2-\sqrt{24}}{2}$

$x_1 \doteq 3.45$ $x_2 \doteq -1.45$